Quantitative Assessment of Risk Perception for Low Dose Risk

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Abstract- Based on the low-dose risk assessment method established in ICRP Pub.26, the log-normal function of annual worker dose was adopted in paragraph 100 of the Recommendations to set dose limits that meet the typical average risk level of safety industries. However, in planned exposure situations where the annual dose approaches the dose limit moderately, it is necessary to add a normal function to the lognormal function to account for dose reduction due to the dose limit. As such a probability distribution, the hybrid log-normal (HLN) distribution was developed in the former JAERI (1980). The genesis mechanism of the HLN distribution has the characteristics of "log + linear increasing/ decreasing" (Kumazawa and Ohashi 1986). On the other hand, cell survival curves with shoulders have "log + linear decrease" characteristic due to the sublethal damage repair effect, and the radiation response per surviving cell have "log + linear increase" characteristic of dose via its logarithmic transformation due to expressing the product of power function and exponential function for dose. As a result, the generalized dose-response relationship, corrected for cell surviving fraction, can be quantitatively modeled based on the HLN genesis characteristics. Thus, the qualitative model of radiation effects and protective risk management can be integrated on the base of the HLN genesis characteristics. In addition, the HLN-generating characteristics are also found in the enzymatic reaction of Michaelis-Menten and the suppression of neutron excess due to increased number of protons in the nucleus, plant growth regulation and reactor power control, and the effectiveness of these management effects becomes apparent in the pure hybridization region bridging between logarithmic and linear scale regions (Kumazawa 2019). Similarly, it is important to adjust the control of "low dose risk increase" associated with "beneficial activity increase" so that the effective result of risk management appears in the pure hybridization region. The HLN genesis characteristics are also found in the temporal and spatial attenuation of air dose rates after the 1FNPP accident (Kumazawa, Toyota, and Kato, 2019) and in the estimated individual dose distributions of evacuated residents (Kumazawa 2020), also applicable to environmental radiation protection. We would like to propose for the future of radiation protection that the concept applying the “log + linear increase” (HLN-generating property) of the gas pedal and brake be included in the system of radiological protection.

Keywords: ICRP Pub.27, individual dose distribution, hybrid log-normal (HLN) distribution, log + linear increase/decrease, quantitative low dose risk sense

1. INTRODUCTION

ICRP Pub.26 (1977) established a system of dose limitation, including dose limits that meet the level of the average annual risk of death in safe occupations (Paragraphs 99,100), as protection against low-dose radiation risks based on ICRP Pub.8 (1966) "The Evaluation of Risks from Radiation". The dose limit for the general public was set to meet the fatal risk acceptable to any individual member of the public (Paragraph 118).

The ICRP Pubs. 60 (1991) and 103 (2007) established a system of radiological protection in which the calculation of general safety risk levels corresponding to dose limits was evolved from the calculation of annual mortality risk to the calculation of lifetime health detriment (cancer mortality and morbidity, hereditary diseases; their resulting loss of life). ICRP Pub.60 recommends that the evaluation of the effectiveness of the actual protection system should be based on the achieved dose distribution and the assessment of each stage of the limiting measures of the potential exposure probability (Paragraph S20). ICRP Pub.103 also recommends the use of dose limits and dose constraints for planned exposure situations and reference levels for emergency and existing exposure situations. For this purpose, we believe it is important to characterize the controlled individual dose distribution in each exposure situation.

Systematic analysis of the characteristics of individual dose distributions, based on a large number of measured dose statistics, should provide a quantitative assessment of the sense of risk in managing low-dose radiation risks. For this purpose, we propose to discuss the characteristics of individual dose distributions in the case of dose reduction management approaching the dose limit, to examine how the sense of risk for low-dose radiation risk is quantified, and to incorporate this quantitative sense of risk be incorporated into future radiological protection systems.

2. INDIVIDUAL DOSE DITRIBUTION

2.1 The “skewed lognormal distribution” problem at doses moderately close to the dose limit

In the field of radiation protection, Gale (1965) introduced the characteristics and use of the lognormal distribution in contrast to the normal distribution, based on the monograph "Lognormal Distribution" (Aitchison and Brown, 1963)、in which a lognormal analysis of the UK AERA annual dose statistics was conducted. As a result, he reported that the lognormal fit was good for doses below about 27 mSv in 1954 (standard 150 mSv) and below 16 mSv in 1963 (standard 50 mSv transition period) , while both were out of the lognormal distribution in the higher dose ranges. (Note: assuming 1 rad = 10 mSv). He explained that the reason for the deviation from the lognormal distribution is that the cumulative doses approaching the annual standard are subject to reduction controls. Although the deviations from the lognormal distribution was small in percentage terms, estimated calculations showed that the deviations were about 70 out of all 2240 persons in 1954 and about 190 out of 570 persons with detectable exposure in 1963. In terms of radiation protection, the exposure tendency (dose distribution) of these workers, which deviated from the lognormal distribution, could not be ignored, but this "skewed lognormal distribution" problem was considered unsolved.

In the 1977 Report of the United Nations Scientific Committee on the Effects of Atomic Radiation (UNSCEAR), the annual dose statistics by occupation collected from various countries were comprehensively analyzed, and a reference distribution based on the lognormal distribution was constructed on the basis of this analysis to fit the intention of the ICRP dose limitation system. The results were cited in ICRP Pub. 26, paragraphs 99 and 100, and had a significant impact on the world, including our own. However, the "skewed log-normal distribution" problem pointed out by Gale (1965) can be seen in many of the results illustrated in Annex E. The reason for the deviation of the occupational dose statistics for US LWRs shown in Figure IV from the lognormal trend (linear trend in the log-probability plot) is given in Annex E, paragraph 55: "There is a control tendency to reduce the dose for annual doses approaching or exceeding 50 mSv.

Thus, in order to solve the "skewed lognormal distribution" problem, radiation protection has reached a stage where the effect of dose reduction controls for doses approaching dose standards/limits should be studied quantitatively, and new probability distributions should be constructed to reflect this effect.

2.2 A probability distribution about the skewed lognormal distribution of doses below limitation

Gale (1965) used the existing important normal and lognormal distributions for distributional analysis of annual dose statistics. The normal distribution was established by Laplace (1820) in a form derived from the binomial distribution and popularized by the error theory of Gauss (1809). As the name "normal distribution" implies, it came to be believed to apply to all data. However, many cases of deviations from the normal distribution were found in vital and social statistics, which were studied as "skewed normal distribution". Among them, Galton (1879) and McAlister (1879) constructed the "lognormal distribution" as a new probability distribution via the geometric mean. The lognormal distribution has been widely used in all fields of social and natural sciences in response to the growing mass-production society of the 20th century.

Further back in time, since the contradiction between the geometric series growth of population and the arithmetic series growth of food production was pointed out in the population theory (Malthus, 1798), the dichotomy of logarithmic variation and linear variation has become the typical cognitive sense regarding quantitative variation. Seen in this framework, the understanding that the random variable that becomes the maximum entropy under the given mean and variance of logarithmic variation is log-normally distributed, and that the random variable that becomes the maximum entropy under the given mean and variance of linear variation is normally distributed, is knowledge in accordance with the above dichotomy.

On the other hand, since the mid-twentieth century of the Club of Rome "Limits to Growth" (1972), with the establishment of governmental agencies for environmental protection, corporate cultures that aim at products emphasizing environmental protection, and today’s emphasis on the SDG (sustainable development goal), in the twenty-first century, moving away from the previous lognormal growth, we are living in era of “skewed lognormal” growth that considers growth limits. This has implications for the future of radiological protection in the face of the “skewed lognormal” problem.

The "skewed lognormal distribution" problem faced in the field of radiation protection cannot be solved from the idea of solving the "skewed normal distribution" problem based on the binary opposition of linear or logarithmic variation. In recent years, radiation protection systems have been viewed as network structures, in which the concept of cybernetics is inevitable. In particular, the feedback function is important to know the danger of a place and to take appropriate measures. Such a function is inherent in the mathematics of worker dose control, and clarification of this function will lead to quantitative clarification of the sense of risk in low-dose risk management.

Gale (1965), in introducing the log-normal distribution in the field of radiation protection based on Aitchison and Brown (1963), introduced the genesis mechanism of the normal and lognormal distributions of randomly progressing biological growth, but did not link this distribution genesis mechanism to the mathematics of dose control for workers. In each stage of the stochastic process showing the genesis mechanism of the lognormal distribution, the predicted dose increment in randomly proportional to the cumulative dose up to the previous stage is placed as $\Delta X_j = \epsilon_j X_{j-1}$, and expressed as the equation of random exposure coefficient $\epsilon_j = \Delta X_j / X_{j-1} \approx \Delta \ln X_j$. based on the central limit theorem (strictly speaking, on the martingale central limit theorem), it is known that the uncertainty sum $\sum_{j=1}^{n} \epsilon_j \to \int_0^T d \ln X(t) \ \propto \ln X_T$ asymptotes to normal distribution with the increase of n. From this, the cumulative dose X_T at the end of the working period [0,T] can be approximated by a lognormal distribution.

However, the control of the reduction of the cumulative dose approaching the dose limit, pointed out in Gale (1965) and 1977 Report Annex E, is to reduce the value of random exposure coefficient ϵ_j at each stage of the exposure stochastic process of the work period by taking measures to reduce the dose rate in the work environment, shortening the working hours, etc. Assuming that the feedback function has the function to further reduce the dose increment as the predicted dose increment is larger, and replacing$\epsilon_j \to \epsilon_j - \rho \Delta X_j$ with $\Delta X_j = \left(\epsilon_j - \rho \Delta X_j\right) X_{j-1}$, it becomes $\Delta X_j = \epsilon_j X_{j-1} / \left(1 + \rho X_{j-1}\right)$ and then $\epsilon_j = \Delta(\ln \rho X + \rho X)$. From this, it can be proved that the uncertainty sum $\sum_{j=1}^{n} \epsilon_j \to \int_0^T d\,[\ln \rho X(t) + \rho X(t)] \propto \ln \rho X_T + \rho X_T$ asymptotes to the normal distribution based on the central limit theorem (strictly speak, on the martingale central limit theorem) (Kumazawa and Ohashi 1986). From this, it can be shown that the cumulative dose X_T at the end of the working period [0,T] becomes a new probability distribution such that $\ln \rho X_T + \rho X_T$ follows a normal distribution.

A few years after the publication of ICRP Pub. 26 (1977), the former Japan Atomic Energy Research Institute (JAERI, the predecessor of JAEA) developed a probability distribution in which $\ln \rho X + \rho X$ is normally distributed with mean μ and variance σ^2, and named it the "hybrid lognormal distribution" (abbreviated HLN distribution) (Kumazawa, Shimazaki, and Numakunai, 1980, 1982; Kumazawa and Numakunai, 1981). The hybrid lognormal distribution was used in the report "Occupational Exposure to Ionizing Radiation in the United States" (EPA 520/1-834-005; Kumazawa, Nelson, and Richardson, 1984), which included an analysis of the effects of dose regulations under the U.S. federal radiation protection since 1960 to 1985 of its prediction. It was also applied to the ALARA analysis of dose control effects at the radiation workplaces in the research reactors JRR-2 and JRR-3 (Kumazawa and Numakunai, 1982; Kumazawa, Matsushita, Yamamoto, and Numakunai, 1988).

The paper on the statistical theory of the hybrid lognormal distribution and the genesis mechanism based on the martingale central limit theorem (Kumazawa and Ohashi, 1986) was cited, and the name "HYBRID LOG-NORMAL DISTRIBUTIONS" was officially published in the Encyclopedia of Statistical Sciences, Supplement Volume (Kotz and Johnson, editor-in-chief, 1989).

2.3 Extension of HLN distribution for dose distribution to other exposure situations

Gale (1965) also referred to the lognormal distribution of random variables with lower and upper bounds, based on Aitchison and Brown (1963), as an extension to other situations in radiation protection. The four-parameter lognormal distribution with $a < x < b$ which is commonly used in the environmental field today, is a probability distribution in which $\ln Y$ follows a normal distribution with mean μ and variance σ^2, where $Y = (X - a)/(b - X)$, and is usually called the Johnson S_B distribution (abbreviated as JSB distribution). For the hybrid lognormal distribution, a probability distribution is also defined in which $\ln \rho Y + \rho Y$ follows a normal distribution with μ and variance σ^2, and is called the hybrid S_B distribution (abbreviated HSB distribution), which corresponds to the Johnson S_B distribution.

The changes in the shape of individual dose distributions, including lognormal (LN) and hybrid lognormal (HLN) distributions, as a function of the degree to which the cumulative dose approaches the dose limit are as follows (Kumazawa, 2008)

Rarely close to limit	LN	$\ln X \sim N(\mu, \sigma^2)$
Moderately close to it	HLN	$\mathrm{hyb}(\rho X) = \ln \rho X + \rho X \sim N(\mu, \sigma^2)$
Considerably close to it	HSB	$\mathrm{hyb}[\rho(X - a)/(b - X)] \sim N(\mu, \sigma^2)$
Extremely close to it	JSB	$\ln[(X - a)/(b - X)] \sim N(\mu, \sigma^2)$

The reference distribution in Annex E of the 1977 UNSCEAR report was a lognormal distribution satisfying the conditions that the mean is 1/10 of the dose limit and the percentage of workers exceeding the limit is 0.1%. If the reference distribution satisfying the same conditions is constructed using a hybrid lognormal distribution, it will be possible to implement protective measures based on an individual dose distribution design in which the dose reduction parameter ρ is selected according to the exposure situation as an appropriate dose distribution form continuously changing from lognormal to normal. If the HLN-type reference distribution design method is combined with the dose distribution design of "LN → HLN → HSB → JSB distribution", which is systematized in relation to the above-mentioned dose limits, it may be possible to apply the method to the prediction and reconstruction of individual dose distributions for various exposure situations as consistent radiation protection engineering (Kumazawa, 2008).

The understanding of individual dose distributions with explicit probability distribution types, as described above, as a risk sense for dose risk management to be conducted by the public in various exposure situations, is considered to be an important issue for the future.

3. APPLICABILITY OF "$\ln \rho X + \rho X$" TO HEALTH PHYSICS

3.1 Potential implications of the function $\ln \rho X + \rho X$

In radiation protection, it is very important to understand the logarithmic variation (random variation of $\ln \rho X$) and linear variation (random variation of ρX). However, as described in section 2.2, in order to solve the "skewed lognormal distribution" problem, it was necessary to introduce mathematics that considers the feedback effect, which is essential in radiation protection. As a result, it was found that the hybrid variability ($\ln \rho X + \rho X$) often plays an indispensable role in radiation protection.

The external random stimulus ϵ, which produces incremental hybrid variability, is transformed via the positive control parameter ρ into logarithmic variability → pure hybrid variability (other than logarithmic and linear variability) → linear variability, through the relationship expressed as $\epsilon \propto \Delta(\ln \rho X + \rho X)$ for an exposed quantity of X. If $\rho \to 0$, it becomes logarithmic variation, and conversely, if $\rho \to \infty$, it becomes linear variation, so the whole becomes a hybrid of logarithmic variation and linear variation, which is called hybrid variation, reflecting as a "log + linear increase".

The hybrid variation is based on the equation $Y = \ln \rho X + \rho X$, and the value of X is scaled on the equidistant number line of Y. Visually, $\rho X < 0.1$ on hybrid scale is considered to be logarithmic scale, and $\rho X > 5$ is linear scale. In the intermediate range of 0.1~5, a purely hybridizing scale region is identified, which is neither a logarithmic scale nor a linear scale. Such a hybrid scale can only be derived based on the concept of integrating both logarithmic and linear variability, breaking away from the dichotomy of the two (logarithmic or linear) since Malthus (1798).

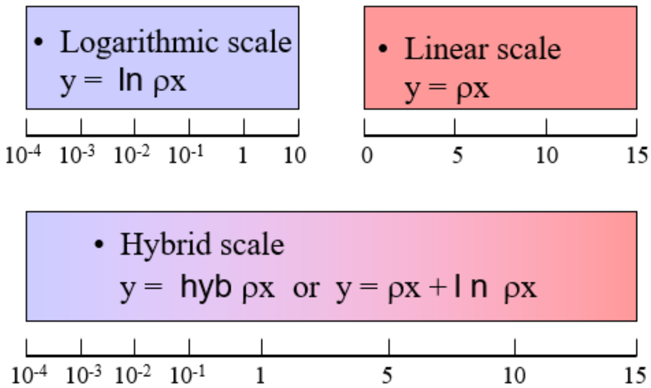


Figure 1 Hybrid scale of ρX

When the external random stimulus ϵ, which is the source of the engine of things, corresponds to the gas pedal of a car, and the control parameter ρ corresponds to the brake, driving a car on a straight road completes the control of risk aversion by operating only the gas pedal and the brake. The hybrid variation equation $\epsilon = \Delta(\ln \rho X + \rho X)$ is equivalent to the following risk management equation that gives the reasonable degree of risk aversion as the predicted value ΔX.

$$\Delta X = \frac{\epsilon X}{1 + \rho X} \quad (1)$$

This is a mathematical expression for risk management that cannot be simplified any further.

As a validation of Equation (1), the neutron excess NE due to the increase of proton number Z for stable nuclei was investigated for half-lives longer than 5 billion years in the Nuclear Chart, JAEA 2014 (JAEA 2014). Corrected to a positive value of $NE' = NE + 3.4$, the increment was found to be expressed as $\Delta NE' = \epsilon NE'/(1 + \rho NE')$ (Kumazawa,2019). Many kinds of elements are known to be randomly produced in the cosmic evolution by nuclear reactions, supernova explosions, neutron star coalescence, etc. While the neutron excess in stable nuclei is explained in detail in nuclear structure theory, Equation (1) can be regarded as a "risk management mathematics" in which nature (natural laws) achieves nuclear structure stability by randomly stimulating element production as ϵ and suppressing the neutron excess on average ρ.

Thus, a concise summary in the form of Equation (1) from a macroscopic point of view, such as the example of neutron excess with respect to nuclear stability, which is developed in detail in physics theory, is considered to be useful for accurately understanding the variable characteristics of quantities in radiation protection practice.

3.2 Applications and extensions of the function $\ln \rho X + \rho X$

In radiation protection, many examples of the function $\ln \rho X + \rho X$ have been reported a lot in most of individual dosimetry (external, internal), environmental dosimetry (normal, accidental), and radiation risk assessment (dose-response relationship, epidemiology, lifetime risk). Some of the examples are shown below.

While the inverse square law of the distance of radiation from its source is well known, the environmental radiation rate is expressed as $\ln \acute{D}(R) = \alpha + \beta\,\mathrm{hyb}(R/R_b)$ as examples of source distance decay for sky-shine and for environmental neutron-ray from the JCO accident (Kumazawa, 2015), where, R is the source distance, $\acute{D}$ is the dose rate, R_b is a boundary distance from power function to exponential function decay, and α, β are the intercept and slope of the linear model, respectively. This equation is called the hybrid scale (HS) model.

It was confirmed that $\ln \acute{D}(t) = \alpha + \beta\,\mathrm{hyb}(t/t_b)$ can be applied to the time decay of the air dose rate $\acute{D}(t)$ from ^{137}Cs deposited on the ground since the several years after the release from the 1FNPP accident (Kumazawa, Toyoda, and Katoh, 2019), which is decomposed and synthesized into two types of HS models (linear relational model by applying hybrid scale) including the early phase of the accident, can be used to predict the early phase of the accident (Kumazawa, 2020).

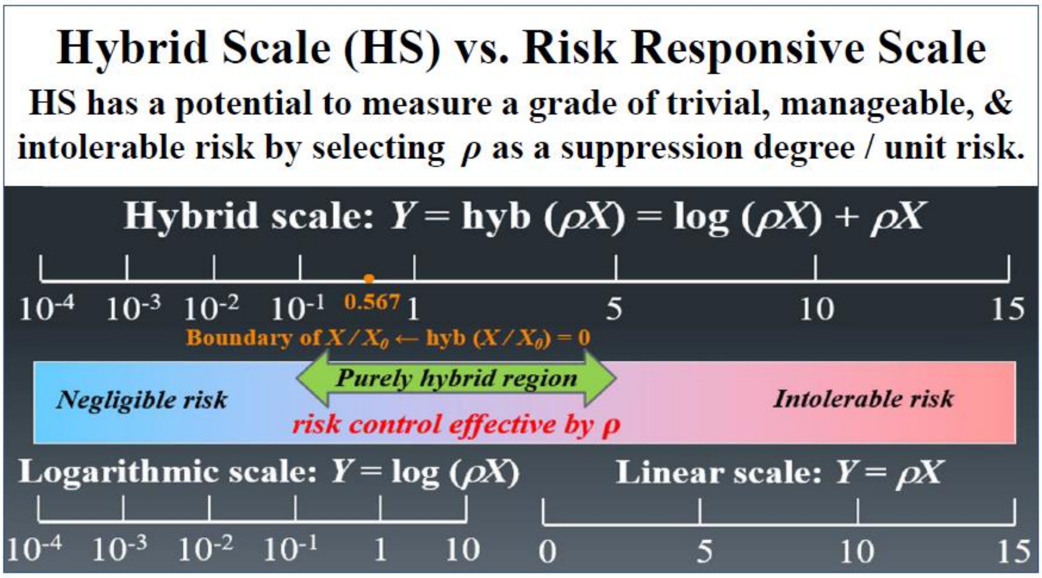
The enzyme reaction theory is expressed by the HS model of Equation (1) and because of the complex involvement of enzymatic reactions in biological responses, the HS model is possibly applicable to dose-response relationships in living organisms. For the split-irradiation experiment (Elkind and Sutton, 1960), the survival fraction S(D), cell inactivation constant λ, and sublethal damage repair effect ρ can be expressed as $dS(D)/\,dD = -\lambda S(D)/[1 + \rho S(D)]$ or $\mathrm{hyb}[\rho\, S(D)] = \delta - \lambda D$, $(\delta = \mathrm{hyb}(\rho), at\ D = 0)$ (Kumazawa, 1994, 2018). The generalized dose-response relationship $I(D) = F(D)S(D)$ is also expressed with HS model by $\ln \mathrm{F(D)}[I(D)/S(D)] = \alpha + \beta\,\mathrm{hyb}(D/D_b)$, which D_b is the boundary dose from the power function increase to exponential increase, and α and β are the intercept and slope of the linear model. This HS model is a good fit (Kumazawa, 2018) for the

X-irradiated mouse translocation fraction (Preston and Brewen, 1973) and the gender-excess relative risk of solid cancer incidence in atomic bomb survivors (Grant et al., 2017). There are more examples.

Since there are many kinds of hybrid variations indicated by the function $\ln \rho X + \rho X$, it is necessary to define a theory to apply them systematically. First, the hybrid variation based on the function $\ln \rho X + \rho X$ forms a hybrid scale consisting of three regions: logarithmic variation region, pure hybrid variation region, and linear region. The second is a two-dimensional hybrid-scale graph paper in which the hybrid scale is applied to the vertical and horizontal sides of a two-dimensional graph paper shown in Figure 2. Details are discussed as a systematic application of the function $\ln \rho X + \rho X$, named hybrid scale (HS) model in the paper (Kumazawa,2019). This extension has a potential to be studied more for the future of radiological protection. Note that Appendix I through Appendix V are useful for understanding the specifics of "$\ln \rho X + \rho X$ ".

4. CONCLUSION AND PROPOSAL

If the variation of the managed dose is viewed as a binary choice between logarithmic and linear variation, the problem of "skewed lognormal distribution" of the cumulative dose approaching the dose limit cannot be solved. However, considering feedback control, which is indispensable for radiation protection, and introducing a reduction factor ρ per unit managed dose, it was proved that the $\ln \rho X + \rho X$, (log + linear increase), yields a hybrid variation of ρX that follows a normal distribution. It was clarified that the basic principle of the radiological protection system is to achieve risk control by effectively activating the dose suppression coefficient ρ (brake) per unit managed dose against the beneficial consequences (gas pedal) foreseen by the random dose-increase stimulus ϵ. It is proposed that this sense of risk for low-dose radiation risk (the pure hybrid variation region shown in the figure below) be incorporated into future radiological protection systems.



Source: Kumazawa (2018, 2020)

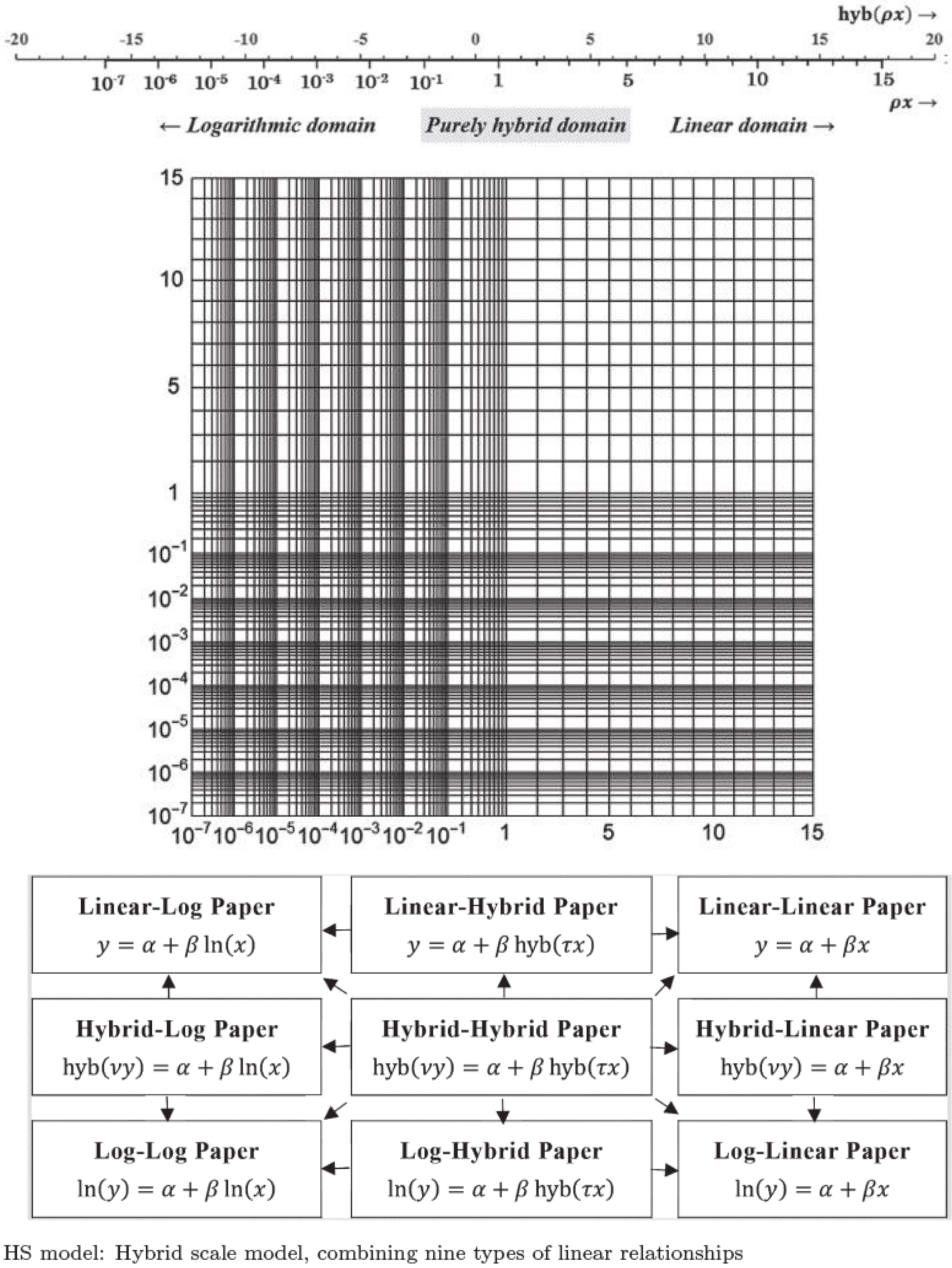


Figure 2 Hybrid Scale, Hybrid-Hybrid Graph Paper (middle) and nine types of Hybrid scale models (lower), providing a linear graph on each of nine domains of hybrid-hybrid graph paper.
cited from Kumazawa (2019).

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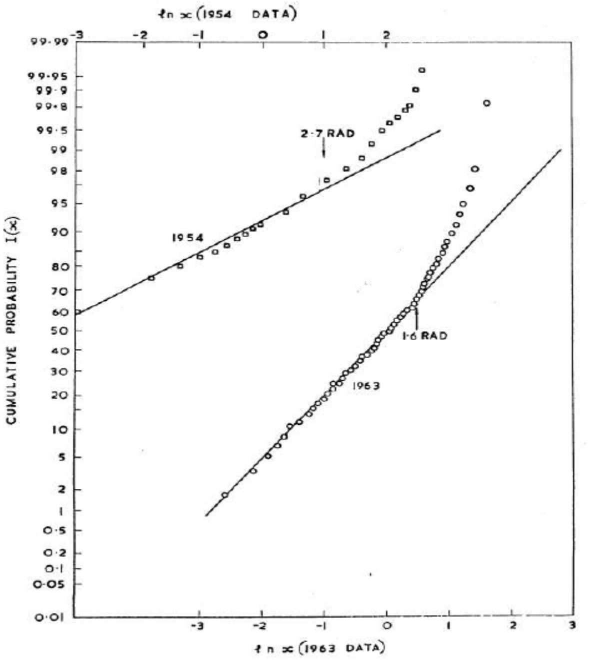


A.E.R.E. R.4736 FIG. 8. DATA ON ANNUAL GAMMA RAY DOSE AT A.E.R.E.

48. As a further example of the distribution of the annual gamma ray dose fig. 8 shows data for 580 workers at A.E.R.E. in 1963. This is must interesting as it shows that for doses less than about l.6 rads the data fit a lognormal line very well, whereas above this value the doses are consistently less than the fitted line predicts. A possible explanation is that a conscious effort is made to reduce the dose received if in the early part of the year the dose is accumulating at a rate which would lead to the derived working limit for annual dose of 5 rads being exceeded. This receives some support from the other data …

Source: Gale (1965)

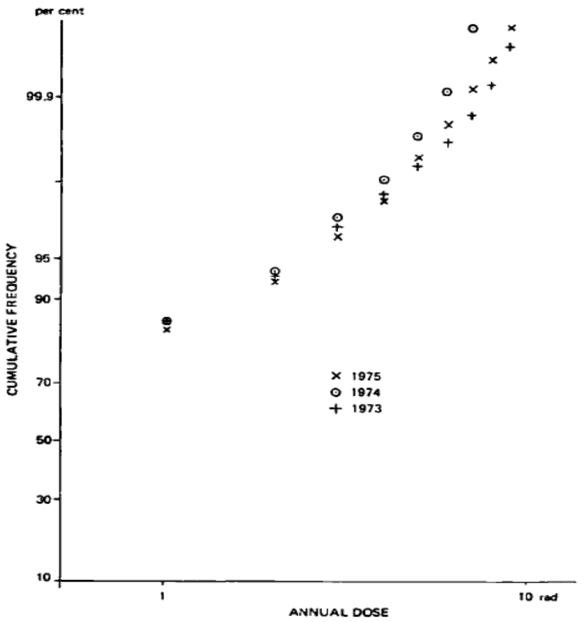


Figure IV. Log-probability plot of annual doses to workers at light-water reactors in the United States, 1973, 1974 and 1975

55. Annual average doses to individuals decreased again in 1974 from the peak in 1972 and remained steady in 1975. The mean number of personnel per plant also showed a decrease in 1974 from the very high figure for 1973, but showed a rise again in 1975. Table 5 shows the annual average doses from 1969-1975 at all United States light-water reactors (78), and figure IV is a log-probability plot of the annual doses (78) for the years 1973, 1974 and 1975. The agreement with a log-normal distribution is not very good, presumably due to a tendency to reduce annual doses approaching or exceeding 5 rad.

Source: Annex E of UNSCEAR 1977 Report

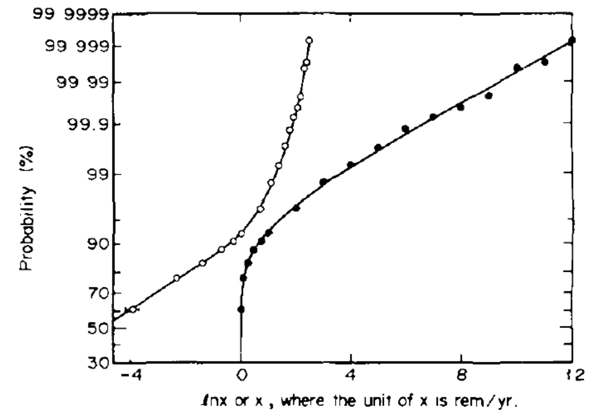


FIG. 5. Normal and lognormal probability plots of annual doses to workers at the licensed facilities of NRC in the United States, 1974. The solid and open circles represent normal and lognormal plots, respectively.

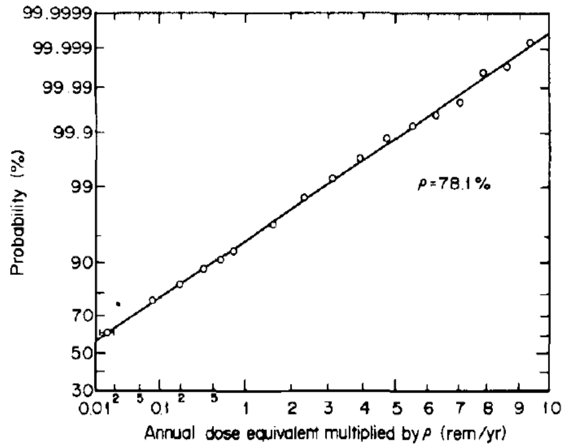


FIG. 6. Hybrid lognormal probability plot of annual doses to workers at the U.S. NRC licensed facilities, 1974.

Source: Health Phys. Vol.41, No.3 465-475 (Kumazawa and Numakunai, 1981)

Appendix II Some examples of HLN analysis From U.S. Occupational exposure (EPA 520/1-84-005)

(Kumazawa, Nelson, and Richardson, 1984)

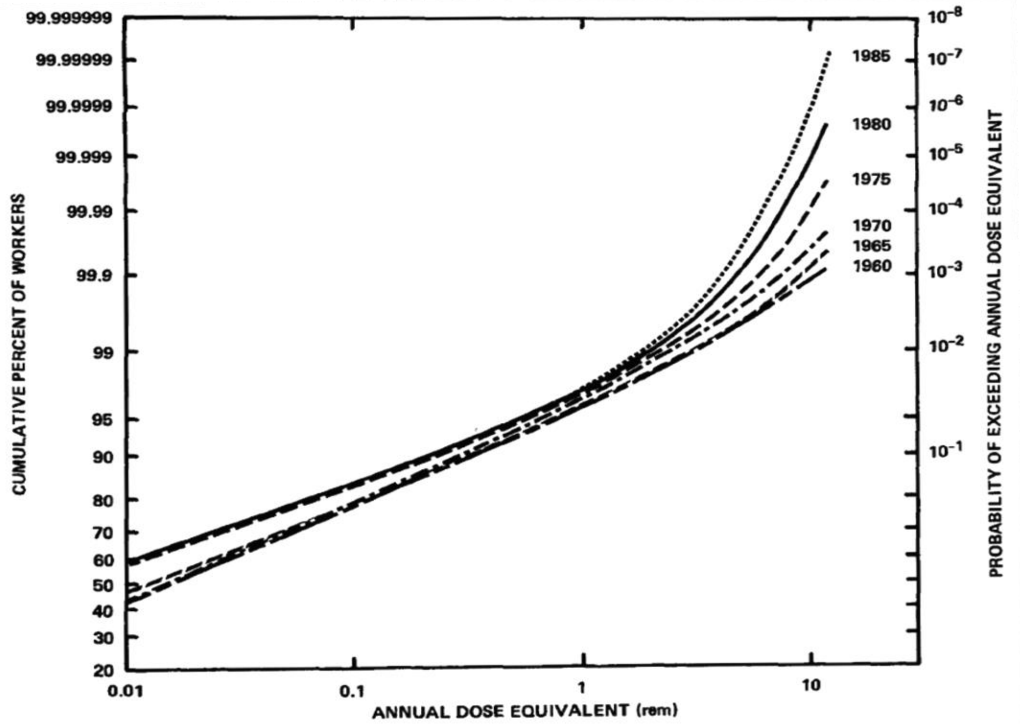


Figure 12. Dose distributions for potentially exposed workers, 1960 to 1985.

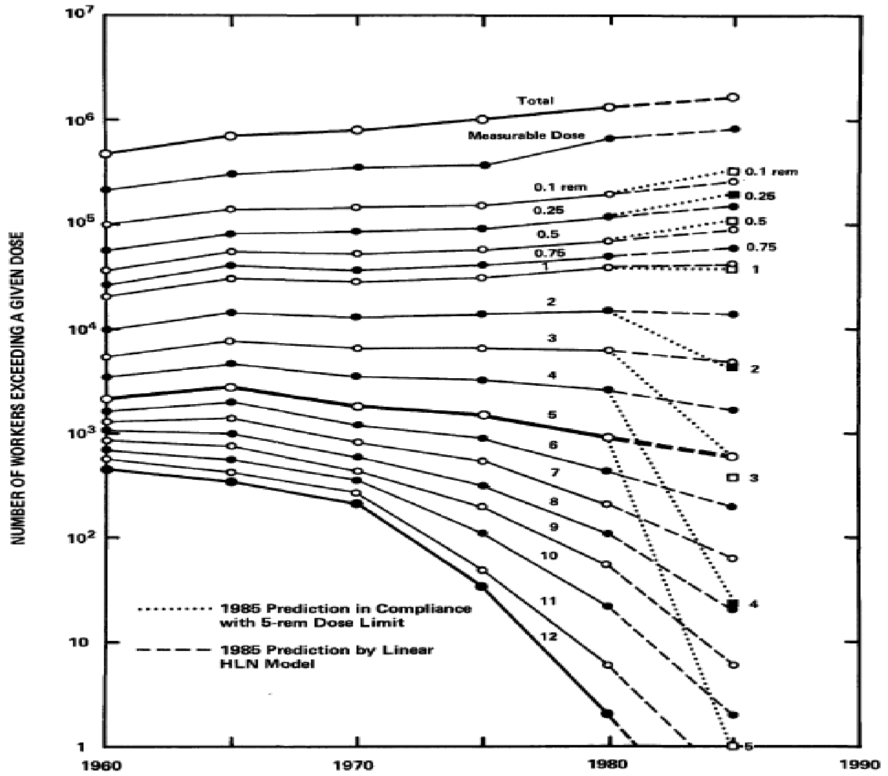


Figure 15. Number of potentially exposed workers exceeding a given dose, 1960 to 1985.

Appendix III HLN analysis of workers doses in special jobs at JRR-2 and JRR-3 cited from IRPA 7 (Sydney)

(Kumazawa, Matsushita, Yamamoto, and Numakunai, 1988)

The hybrid log-normal distribution is applied to the distribution of doses X so that $Y=\ln\rho X+\rho X$ $(\rho>0)$ is normally distributed with parameters (μ, σ^2). The HLN distribution function, $\Omega(x)$, is given by

$$\Omega(x) = \int_0^x \frac{1}{\sqrt{2\pi}\,\sigma}\left(\frac{1}{x}+\rho\right)\exp\left(-\frac{\{\ln\rho X+\rho X-\mu\}^2}{2\sigma^2}\right)dx \qquad (1)$$

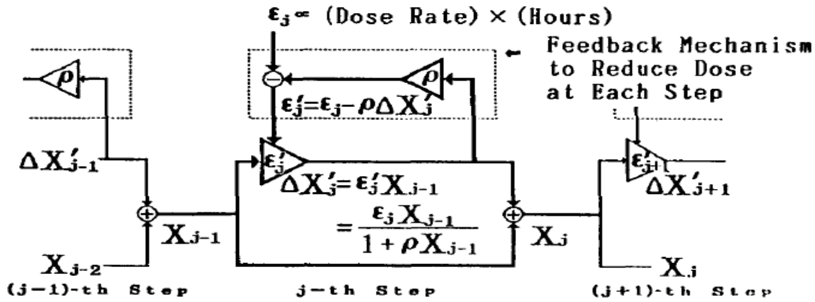


Fig. 1 A mathematical model for the control process of dose reduction

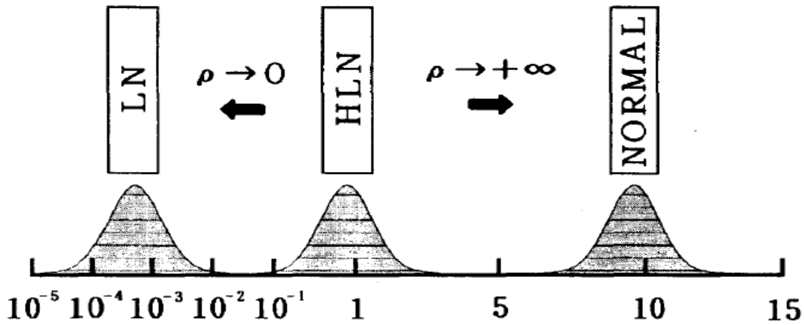


Fig. 2 Hybrid scale: hyb(t) = lnt+t. Putting $t=\rho X$, X changes from a log scale to a linear scale according to ρ.

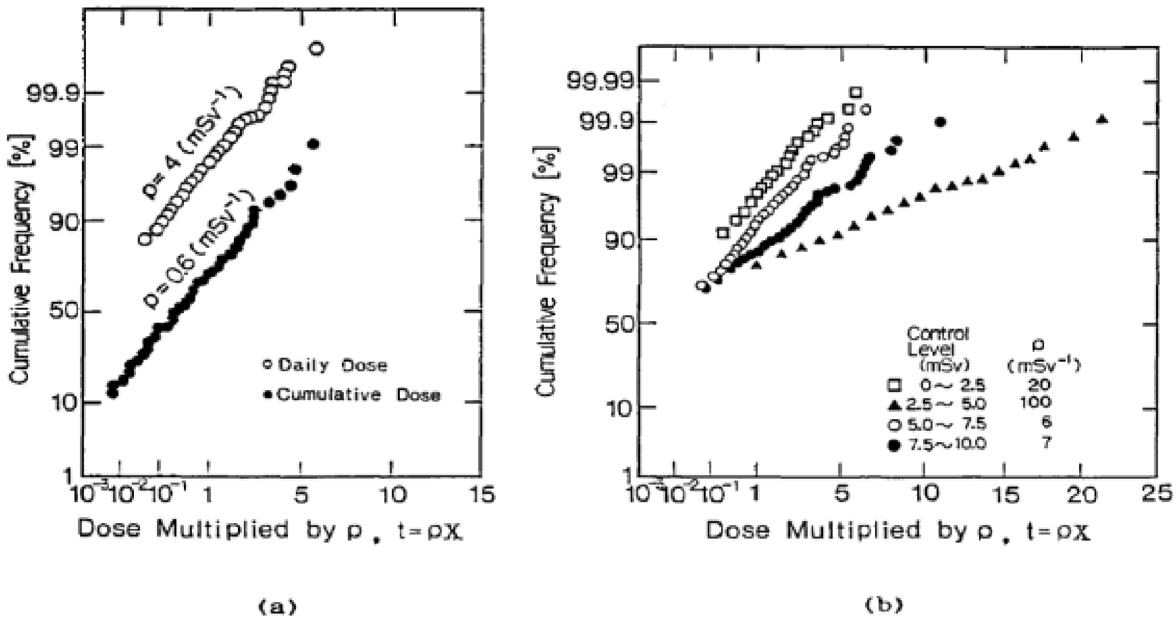


Fig. 4 HLN probability plots of daily and cumulative doses at JRR-3:
(a) daily doses and cumulative doses over the whole period,
(b) four groups of daily doses according to the range of control levels of dose accumulation applied to workers.

Appendix IV A historical review of occupational dose distribution

(Kumazawa, 2002)

IAEA-CN-91/118

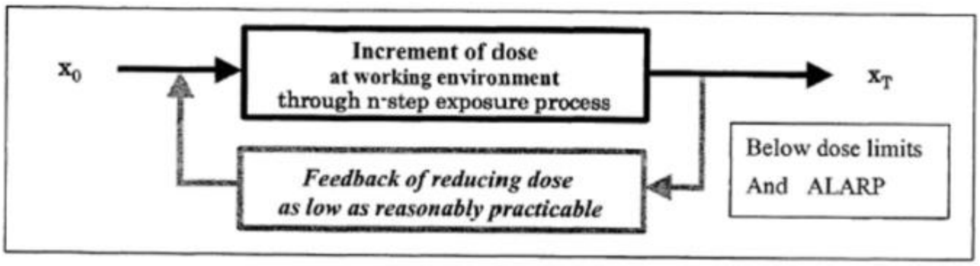


Figure 1. A hybrid model of reducing dose as low as reasonably practicable

We need to know: '*How do we effectively invest our resource of radiation protection to reduce dose in the range of low dose or high dose via feedback effort?*' It is natural that the investment in dose control is stronger at high dose than at low dose. In Equation (1) a controllable quantity is the random intensity of exposure ε_k and the control of reduction should be based on the magnitude of increment Δx_k. Then the exposure intensity ε_k is replaced by the reduced intensity $\varepsilon_k - \rho\Delta x_k$ of a negative feedback control of dose reduction with feedback factor of ρ mSv^{-1} by means of shorting working time, setting more shielding, etc. The exposure process to obey the law of proportionate effect with feedback control is defined as follows, where the hybrid function is defined as hyb(x)=x+ln(x):

$$\Delta x_k = \varepsilon_k x_{k-1} \ , \ \Sigma\varepsilon_k = \Sigma\Delta x_k / x_{k-1} = \Sigma\Delta \ln(x_k) \approx \ln(x_T) - \ln(x_0) \quad (1)$$

$$\Delta x_k = \varepsilon_k x_{k-1} = (\varepsilon_k - \rho\Delta x_k)\, x_{k-1} \ \therefore\ \Delta x_k = \varepsilon_k x_{k-1} / (1+\rho x_{k-1}) \quad (2)$$

$$\Sigma\varepsilon_k = \Sigma(\rho\Delta x_k + \Delta x_k / x_{k-1}) = \Sigma\Delta\{\rho x_k + \ln(\rho x_k)\} = \Sigma\Delta \mathrm{hyb}(\rho x_k) \approx \mathrm{hyb}(\rho x_T) - \mathrm{hyb}(\rho x_0) \quad (3)$$

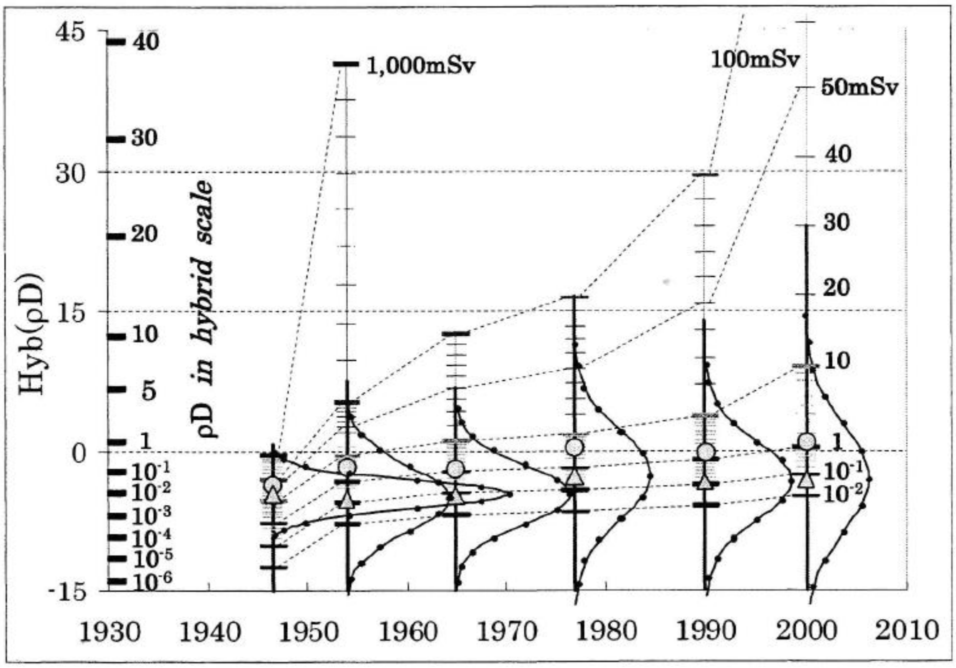


Figure 2. Historical scales of individual doses controlled in accordance with the ICRP Recommendations. Data: 1943-1950 (plot: 1946) UK medical workers [7]; 1954 AERA personnel [8]; 1965 ORNL personnel [9]; 1977 and 1990 NRC licensed-LWR workers [5]; 2000 Japanese nuclear workers [10]. Circles represent mean dose and Triangles are median dose. Bold vertical lines represent the range of actual individual doses.

ON A HYBIRD SCALE MODEL OF DOSE-RESPONSE RELATIONSHIPS UNIVERSALLY APPLIED TO VARIOUS DATA OF IONIZING RADIATION EXPOSURE

SHIGERU KUMAZAWA, Formerly JAERI

OBJECTIVE

To evaluate the low dose risk, this is to develop a universally applied method for dose-response data with a hybrid scale (HS) model that integrates multiplicative and additive reactions.

METHOD

Generalized Hybrid Scale (GHS) Model

Incidence $\log[I(D)] = \log[F(D)] + \log[S(D)]$

HS Model of F(D) = I(D) / S(D)

$\log[F(D)] = \alpha + \beta\ \mathrm{hyb}(\tau D)$

HS Model of S(D), cell survival

$\mathrm{hyb}[\rho S(D)] = \delta - \lambda D$, $\delta = \mathrm{hyb}(\rho)$

$\log[S(D)] = \rho[1-S(D)] - \lambda D$

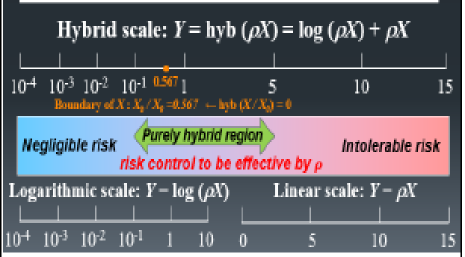
α, β : model parameters

τ : effect modifier per dose

ρ : feedback factor of sublethal cell repair

λ : inactivation constant per dose

Hybrid Function: $\mathrm{hyb}(\rho X) = \rho X + \log(\rho X)$

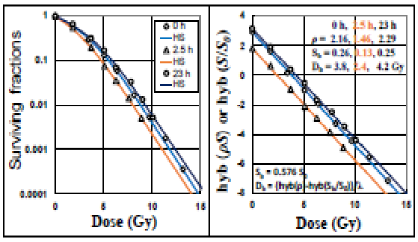


Dose reducing	$\Delta D_i = \varepsilon_i D_{i-1}/(1+\rho D_{i-1}) \rightarrow \rho\Delta D_i + \frac{1}{D_{i-1}}\Delta D_i = \varepsilon_i$ $\Delta\{\rho D_i + \log(\rho D_i)\} = \Delta\,\mathrm{hyb}(\rho D_i) = \varepsilon_i$
Cell repairing	$dS/dD = -\lambda S/(1+\rho S) \rightarrow \left(\rho + \frac{1}{S}\right) dS/dD = -\lambda$ $d\{\rho S + \log(\rho S)\}/dD = d\,\mathrm{hyb}(\rho S)/dD = -\lambda$

The concept of hybrid scale is important to identify the effective range of risk control for radiation protection and bio-defense system.

RESULTS-1 S_HS model applied to data of Elkind and Sutton (1960)

HS model: $\mathrm{hyb}(\rho S) = \mathrm{hyb}(\rho) - \lambda D$

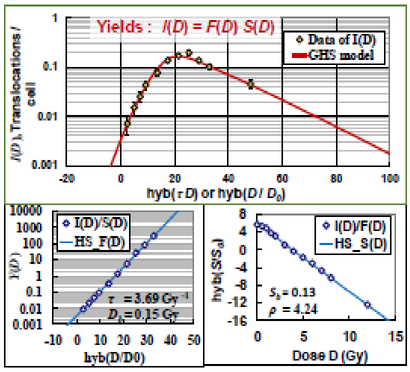


Fraction of surviving mammalian cells by split-dose of x-ray. The fitting is all good and ρ is small for 2.5 h due to less repair.

S(D): Cell survivals	F(D): Yields / viable cell
$\frac{dS}{dD} = \frac{-\lambda S}{1+\rho S}$	$\frac{dF}{dD} = \beta F\left(\tau + \frac{1}{D}\right)$
$\mathrm{hyb}(S/S_0)$ vs D	$\log(F)$ vs $\mathrm{hyb}(D/D_0)$

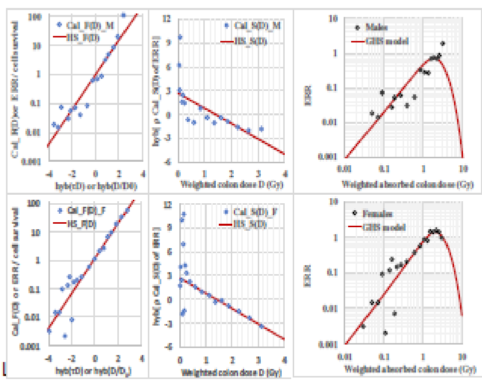
RESULTS-2 F_HS model, GHS model applied to Preston & Brewen (1973)

HS model: $\log[F(D)] = \alpha + \beta\mathrm{hyb}(\tau D)$



Reciprocal translocations of mice spermatogonia by x-ray. The fitting is good and a bio-system of F(D) is normal for $D < D_0$.

RESULTS-3 GHS model (Grant et al., 2017)



females at attained age of 70 years after exposure at age 30 years. The fitting is all good over the dose range of S(D), F(D) and I(D) with a similar characteristics between males and females for extrapolating the low dose-response. The GHS model is better than L or LQ model due to using all available data.

CONCLUSION

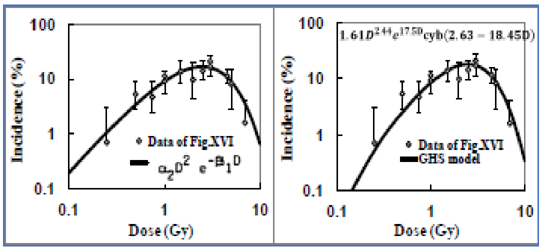
1. The HS model of survival S(D) was confirmed on data of Elkind and Sutton (1960).

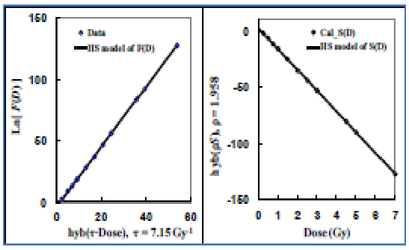
2. The GHS model was fitted well to data of Preston and Brewen (1973), Mole (1984) and Majo et al. (1986), others.

3. The GHS model was fitted well to LSS solid cancer incidence ERR (Grant et al. 2017), better than L or L-Q model fitted to data in the range over 0.005 to 1 Gy or > 4 Gy.

REMARKS:

- Source: From Figure XVI, ANNEX B, UNSCEAR 1986 REPORT
- Data: Myeloid leukemia incidence of male CBA mice to x-rays (Mole, 1984; Majo et al., 1986)
- Results: The GHS model fitting is good and it predicts a smaller risk coefficient in the low dose range than the model shown in Figure XVI of the UNSCEAR 1986 Report.
- Transformations per surviving cell (Borek, 1984) in Figure VII of the ANNEX B is also fitted by HS model of F(D) well.





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